Bulletin of the Swiss Statistical Society,
Nr. 14, March 2013, p. 4-10, Bern.

Johann Heinrich Lambert: An Admirable Applied Statistician

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26 February 2013

1 Introduction

Johann Heinrich Lambert is best known for his work in physics. The law of Bouguer-Lambert-Beer on the absorbtion of light may be his most famous achievement. He is also well known in mathematics, for example for his proof of π and e being irrational numbers, for his work on non-additive probabilities and for his contribution to non-Euclidean geometry. He worked on numerous problems of natural science, for example in astronomy and in cartography (Lambert-projection). Last but not least, Lambert was a philosopher with a keen interest in epistemology. However, his work in statistics has long gone unattended. But Lambert was an eminent applied statistician and this in a very modern sense of statistical data analysis. His studies of measurement and how to gain insight into the laws of nature in spite of measurement error lead him to formulate the principle of maximum likelihood and to devise a rule for treating outliers. In addition, he wrote about fitting a line to bivariate measurements and developed a theory of errors.

Several aspects may have hindered the recognition of Lambert as a statistician. First, the discipline of statistics did not exist at that time. Probability theory was established by Jacob Bernoulli in 1713. But when Lambert published his masterpiece, the Photometria, in 1760 statistics did not exist. Second, Carl Friedrich Gauss developed the normal distribution, showing that the arithmetic mean is the optimal estimator under the normal distribution and established the least squares principle in 1795. Thus the work of Lambert was overshadowed by the work of

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Gauss, though Gauss probably built on Lambert's work. Third, Lambert wrote in Latin because as a person without an academic education he probably wanted to demonstrate that he was on a level with the science of his time. Unfortunately the German translation of the Photometria by E. Anding in 1892 (130 years later!) left out the paragraphs where Lambert stated the maximum likelihood principle.

A translation from Latin of these paragraphs is now available in English thanks to Illuminating Engineer, David DiLaura (Lambert, 2001). In other words, it needed 240 years until the invention of the maximum likelihood principle was translated into a modern language.

In this article some light is shed on the life and the merits of Johann Heinrich Lambert as an applied statistician. It is not possible to do justice to Lambert here, first because the author is not a historian and second because Lambert was highly productive in many diverse areas such that a full appreciation of his life and work is beyond the scope of this article. Most of the material cited here is based either on the original work of Lambert, mainly (Lambert, 1760) and (Lambert, 1765) and on the English translation by DiLaura (Lambert, 2001). Section 2 provides a short overview over Lambert's life. Section 3 shows some of the statistical highlights of Lambert and Section 4 argues for the importance of Lambert as an applied statistician.

2 Life of Johann Heinrich Lambert

Johann Heinrich Lambert was born on 26 August 1728 in Mühlhausen (today Mulhouse, France). Mühlhausen was at that time associated to Switzerland. Lambert received six years of formal education from the municipality but had to leave school to help his father, a tailor, when he was 12 years old. However, Lambert never stopped learning though he did not attend any formal school afterwards. He studied French, Latin and Mathematics largely on his own. He became an assistant to the city clerk of Mühlhausen, J. H. Reber, then a bookkeeper to an industrialist and finally in 1746 a secretary to Prof. J. R. Iselin in Basel. In this position he gained access to the knowledge of physics and mathematics of his time.

In 1748 he obtained a position in Chur as a private tutor to a grandson of Count Peter von Salis. At the court of von Salis, Lambert could finally pursue his research on physics and optics. The little portrait in Figure 1 may date from that time. He travelled with his pupil through Europe, meeting many eminent scientists and continuously pursuing his research. He became a member of the "physikalisch-mathematische Gesellschaft" of Basel in 1754. From 1759 Lambert travelled on his own through Europe. During that time he published his early masterpiece, the Photometria (Lambert, 1760).

Lambert was living in rather poor conditions, though he received some support

Figure 1: Johann Heinrich Lambert

from the academies he was a member of. After long deliberations and in spite of Lambert's eccentric character he became a member of the Royal Academy of Berlin in 1765. Finally Lambert had a secure post and he started researching and publishing on a diverse topics of his interest. In this time of high productivity he proved that π and e are irrational, wrote about philosophy, studied non-additive probabilities and made contributions to hyperbolic functions and to cartography. Lambert died in Berlin on 25 September 1777.

3 Lambert the Applied Statistician

Lambert was an applied statistician. He was firmly convinced that any law should be tested empirically. Thus, after developing a method to determine the content of barrels, he says "All methods are then tested by true real experiments involving many types of barrels ..." (Lambert, 1765, Foreword). And he developed statistical methods based on his experience with experimental conditions. Thus he argues that the observation with the largest deviation from the mean should be omitted from the mean (see (Lambert, 1760, p. 136) or (Lambert, 2001, p. 99).

Lambert introduced the principle of maximum likelihood when explaining his experiment VI in the Photometria (Lambert, 1760, p. 125). In experiment VI Lambert determines the distance between the center of the reflection of a candle on a wall and the point where the brightness visibly diminishes. He argues that many error sources may influence this distance. He provides arguments why he assumes the distribution is symmetric and concludes that the mean is the best way to estimate the true distance.

He repeats the experiment five times and, as explained above, would leave out the experiment with largest deviation from the mean. In his Section 287 Lambert (1760) argues that "a single notable error disturbs the mean quantity more for a long time, unless by increasing the number of experiments another equally notable but negative error is added, which thus cancels the first" (Lambert, 2001). Lambert argues that leaving out the largest deviating observation the variability is smaller because the arithmetic mean of the remaining observations moves closer to the true mean (Lambert, 1760, Section 290). Thus Lambert argues from his insight into the nature of imprecise measurements that one-sided trimming of one observation of five is a sound statistical method. It needed a long time of statistical development until with the work on robust statistics trimmed means came into fashion again and received a theoretical underpinning.

Lambert assumes that the deviation from the true value has a distribution which is not uniform. Actually he assumes that the support of the distribution is finite and gives a theoretical argument why for his experiment smaller distances from the true value occur more frequently than larger ones. He describes the distribution as shown in Figure 2. The distribution is somewhat more platikurtic than the normal. However, Gauss developed the normal distribution later only.

In Section 296 of the Photometria Lambert argues that there is an observed frequency distribution and a true frequency distribution of the observations. Moving the location of the observed frequency distribution along the real line until a coincidence with the true frequency distribution is reached must show the true location. Lambert goes through a derivation of the possible samples coinciding with observed frequencies of particular values and arrives at the following statement in Section 300: "Fixing the number of observed chances n, m, l, k, the coefficient which is derived from the permutations [the number of possible cases] will be constant, whence it will be $N \propto p^m q^n r^l s^k$[1]. And so the number of possible cases will be the product of the true chances raised to the powers which are equal to the observed chances." . And later in Section 303: "Since in general that case

[1] Our notation, where p, q, r, s are the true probabilities PN, QM, RL, SK

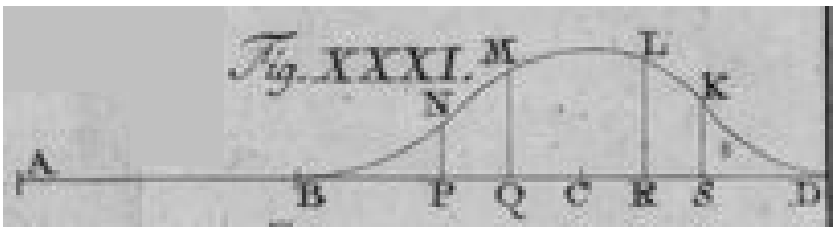
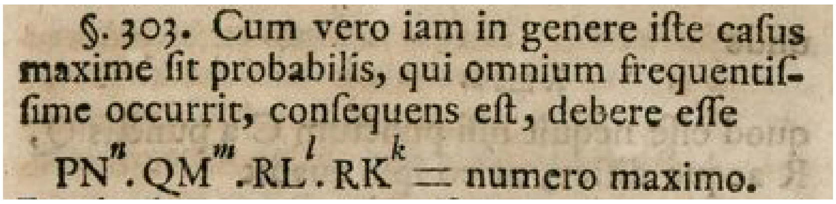


Figure 2: Figure XXXI reproduced from Lambert 1760



§. 303. Cum vero iam in genere iſte caſus maxime ſit probabilis, qui omnium frequentiſſime occurrit, conſequens eſt, debere eſſe

$PN^n . QM^m . RL^l . RK^k =$ numero maximo.

Figure 3: Lambert's Maximum Likelihood Principle (Lambert, 1760).

is most probable which occurs most frequently of all it should be that $p^m q^n r^l s^k$ equals the greates number" (Lambert, 2001). This is the Maximum-Likelihood principle! Lambert's original text is shown in Figure 3 (Note the obvious typographic mistake RK instead of SK). Lambert then shows the log-likelihood, its derivative and finally the maximum likelihood equation (Lambert, 1760, Section 304).

This amazing achievement went unnoticed for a long time. The German translator E. Anding of Lambert's work, who was mainly interested in the foundations of physics of light by Lambert, thought that the statistical Sections 271 to 306 were of little importance and decided to omit them (Lambert, 1892, p. 94). This means that Lambert's ideas about Maximum Likelihood were not translated into a modern language for a long time, probably until 2001. Only a few historians, like Sheynin (2009), recognised the value of Lambert's thoughts. The original writing in Latin may have been necessary to establish Lambert as a scientist because he had no academic training at all. But it may also be a reason why Lambert's work as an applied statistician did not receive the attention it merits.

Lambert also investigated how a physical law can be demonstrated. He saw the importance of appropriate transformation to linearity and he derived his way of fitting a line. He wrote in German in his work "Beyträge zum Gebrauche der Mathematik und deren Anwendungen" : "Die Tabelle sollte so herauskommen, dass die sämtlichen Observationen am wenigsten davon abweichen." (Lambert, 1765, p. 428) (In English: The table should result in such a way that all observations should deviate least from it). Lambert describes a method of line fitting, which should achieve this objective. It uses the bivariate means of the upper and lower half of the data according to the values on the abscissa (x-values) and derives the line from these two points (Lambert, 1765, p. 437). Lambert proceeds to leave out from this calculation the observation with the largest residual and uses the resulting change of the estimates as a measure of their reliability. In modern terms, Lambert used the influence of the observation with the largest residual as a measure to judge the variability. Using all observations he would have obtained a Jackknife variance estimate. Obviously this would have demanded a computing power which was not available at that time and which Lambert himself might deem unnecessary since he was only interested in describing the precision up to a point where a physical law could be established.

4 Final Remarks

Johann Heinrich Lambert was a universal genius to whom the scientific community owes many important discoveries. This is all the more remarkable as Lambert only went to school until he was 12 years old. He taught himself mathematics, physics, astronomy, cartography and philosophy.

The career of Lambert from a tailor's son to a member of the Royal Academy of Berlin was remarkable. He must have been vested with iron tenacity to pursue his research. On the other hand his character may have been difficult at times, as the long process of his election to the Royal Academy or his disagreement with a nomination at the Bavarian Academy of Sciences and Humanities showed. Nevertheless, his example may well serve as an inspiration to young scientists.

His range of interests was all embracing and compared with the narrow field of modern scientists nearly universal. The only professionals he seemed to profoundly mistrust were medical doctors and maybe his early death at 49 was due to this mistrust. However, it also shows his deep conviction that only empirical science, from which medicine at that time was still rather far removed, is to be trusted. Openness to all questions of discovering knowledge was a basic attitude of Lambert and in that sense he is a worthy representative of his age of enlightenment.

As a statistician Lambert worked from the design of experiments and their philosophical foundations through the execution, the data collection, the statistical methods up to the theory he needed. His objective was to develop the tools to derive knowledge from his observations. It may be a mistake that he did not develop the theory further than needed for his purpose but it is also the strength of his theoretical developments to remain rooted in practical experiments. Due to his way of tackling the practical problems of statistics and due to his scientific achievements in statistics Johann Heinrich Lambert merits being considered a shining example of an applied statistician.

Acknowledgement: I thank Brigitte Sprenger and Victor Panaretos for their help and comments.

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